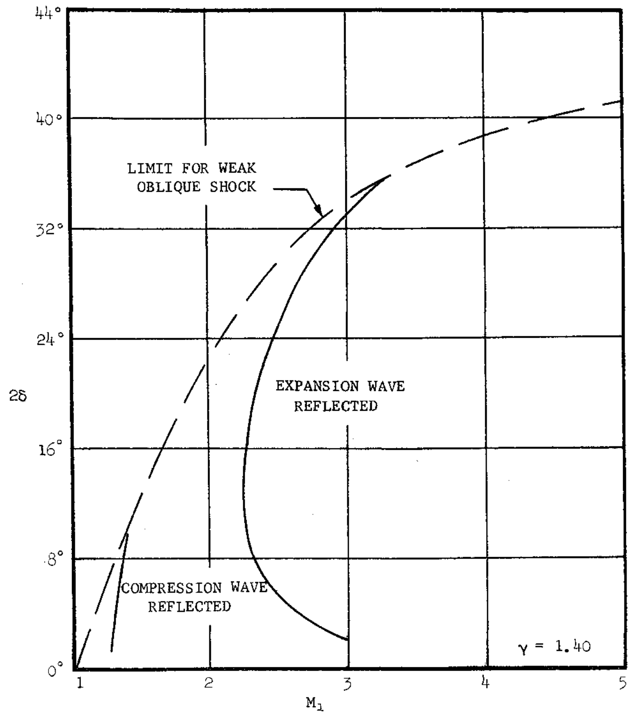

Fig. 3 Condition for no-reflected wave from the shock intersection (γ = 1.40).

values of M_1 will produce a reflected expansion wave, and lower values of M_1 will produce a reflected compression wave. The inverse obviously holds for negative signs.

Figures 2–4 show as solid lines the resulting no-reflection conditions and the regions of each type of reflected wave. It may be seen that two values of M_1 meet the no-reflection requirements at low total turning angles. Only one solution exists at moderate angles, and no solution exists for high angles. Likewise, for a given value of M_1, either two, one, or no values of total turn angle may meet this requirement. Changes in γ give no qualitative change in the result.

The numerical data from which the graphs are plotted are available in the M. S. report of the junior author[2] and are obtainable from the School of Mechanical Engineering, Oklahoma State University.

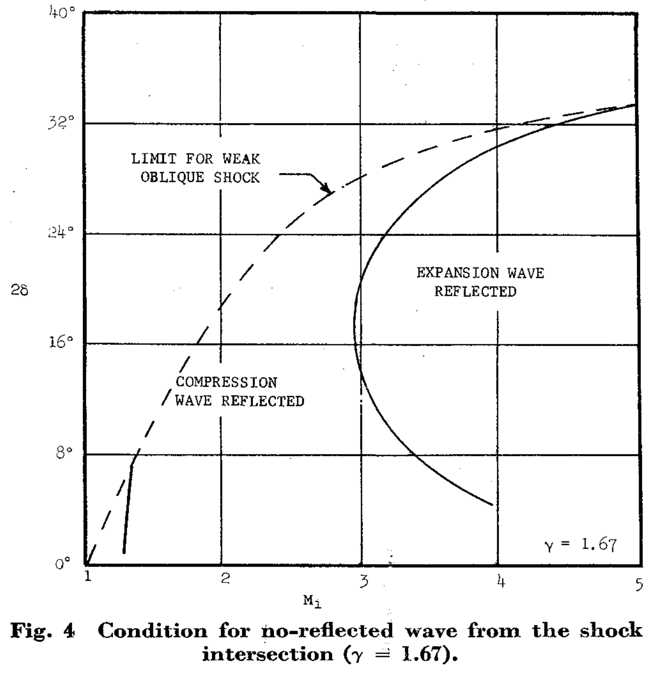

Fig. 4 Condition for no-reflected wave from the shock intersection (γ = 1.67).

References

[1] Thompson, M. J., "A note on the calculation of oblique shock-wave characteristics," J. Aeronaut. Sci. **17,** 741–744 (1950).

[2] Flynn, J. J., Jr., "Investigation of slip-line interaction for two coalescing oblique shocks," M. S. Report, School Mech. Eng., Okla. State Univ., Stillwater, Okla. (January 1963)

Recombination Losses in Rocket Nozzles with Storable Propellants

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The advent of space rocket engines using low chamber pressures and high area ratios has focused attention on the loss in I_{sp} due to incomplete recombination of free radical species during nozzle expansion. Analyses are available[1–3] to determine the losses once the chemical kinetics and their reaction rate constants are known. The kinetics for propellant combinations such as hydrogen-oxygen[2] and RP-1 oxygen[4,5] are fairly well known. However, recombination data for storable propellant combinations such as nitrogen tetroxide-mixed amines have not been available. This note presents a method of accounting for recombination losses for such storable combinations.

Inspection of the species present in rocket combustion chambers, shown in Table 1, using the propellants nitrogen tetroxide in a 50-50 by weight mixture of unsymmetrical dimethyl hydrazine and hydrazine reveals the following facts:

1) The free radical or "active" species present which can recombine in the nozzle to yield thermal energy include CO, H, OH, NO, and O.

2) The number of moles of these species decrease with increasing pressure. The decrease is especially rapid for the species O, OH, and H.

3) The number of moles of molecular nitrogen is large. The amount of nitrogen in other nitrogen-containing fragments is quite small. Thus, nitrogen reactions will not affect the I_{sp} recoverable in the nozzles due to the relatively small number of moles of non-N_2-nitrogen species and the fact that the energy levels of their recombination reactions are low.[6]

Therefore, the main heat-producing reactions in the nozzle are the following:

$$H + H + M \rightleftharpoons H_2 + M$$

$$OH + H + M \rightleftharpoons H_2O + M$$

$$O + O + M \rightleftharpoons O_2 + M$$

$$CO + O + M \rightleftharpoons CO_2 + M$$

These species are the same as those present in the combustion of RP-1-oxygen and hydrogen-oxygen. The chemical kinetics for these reactions are fairly well known. Thus, the kinetics of the C-H-O system can be used in the case of combustion of nitrogen tetroxide-amine combinations.

An approximate analysis has been developed by Kushida[2] and Koppang et al.[4] using the freezing-point criterion of Bray for the C-H-O system. Kinetic principles allow one to use the freeze-point relaxation criteria of Bray for atomic recombination and one-dimensional shifting equilibrium data to estimate nonequilibrium performance of the forementioned propellants with the following assumptions:

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Table 1 Composition of combustion chamber products; N_2O_4/50% UDMH — 50% N_2H_4; *MR* = 2.0

	Pc = 100 psia Moles	*Pc* = 1000 psia Moles	*Pc* = 10,000 psia Moles
CO	0.3295	0.3032	0.2736
CO_2	0.2252	0.2514	0.2808
H	0.1082	0.0553	0.0255
NHO	...	0.0070	0.0125
OH	0.2063	0.1582	0.1055
H_2	0.3326	0.2734	0.2238
NO	0.0490	0.0470	0.0389
O	0.0400	0.0185	0.0068
O_2	0.0817	0.0475	0.0223
H_2O	1.6596	1.7660	1.8539
N_2	1.4974	1.4949	1.4961
Avg molecular wt	22.07	22.61	23.04
T_c,°R	5632.9	6035.2	611.2
$(T_c/\text{molecular wt})^{1/2}$	16.00	16.34	416.68

1) The three body recombination reactions are controlling from both a kinetic and energy release viewpoint.[2]

2) The important three body recombination rate constants are approximately equal.[2, 5]

3) The oxidation of carbon monoxide is fast compared to the three body reactions and is rate controlled by the water-gas reactions.[4, 7]

$$CO + OH \rightleftharpoons CO_2 + H$$

$$OH + H_2 \rightleftharpoons H_2O + H$$

(This assumption and the previous statements on nitrogen species implies that essentially the kinetics are those of H_2-O_2. Other species act only as diluents or third bodies.)

4) The equilibrium parameters may be calculated from one-dimensional flow theory.

5) The combustion process is 100% efficient.

By selecting the fastest three body reaction, assumptions 2 and 3 may be made optimistic (they predict better performance than really should be expected). Assumption 1 is highly accurate and assumption 5 is inherently optimistic. Therefore, in general, the results of these calculations are optimistic and worse performance is to be expected in the real case.

Bray[1] and Hall[3] have solved the exact aerothermodynamics equations for atomic recombination and have found that the actual chemical process follows equilibrium down to a station in the nozzle where departure is noticeable. At this point a very short transition time is required for the flow to become chemically frozen and remains so during the remainder of the expansion. Bray has correlated this transition period (referred to as the freeze area ratio) with a semitheoretical relaxation parameter based on equilibrium conditions:

$$B = \frac{\text{equilibrium rate of change of species } i}{\text{rate of change allowed by chemical kinetics}} \tag{1}$$

Application of Eq. (1) and the forementioned assumptions yields

$$\text{chemical scale factor} = C_i = Bkr^*/\tan\theta$$

$$C_{\text{OH}} = \left[\frac{2\epsilon^{1/2}u}{n^2x_{\text{H}}}\;\frac{d\,\ln(x_{\text{OH}}/\bar{m})}{d\epsilon}\right]_{\text{equil}} \tag{2}$$

$$C_{\text{H}} = \left[\frac{2\epsilon^{1/2}u}{n^2(2x_{\text{H}} + x_{\text{OH}} + x_{\text{O}})}\;\frac{d\,\ln(x_{\text{H}}/\bar{m})}{d\epsilon}\right]_{\text{equil}} \tag{3}$$

where

Bk = effective three body recombination rate constant, $(\text{cm}^3/\text{mole})^2/\text{sec}$
m_i = molecular weight of ith chemical species, g/mole
$\bar{m}$ = mean molecular weight, g/mole
n = molar density, moles/cm³
r^* = nozzle throat radius, cm
x_i = mole fraction of ith species
u = local gas velocity, cm/sec
ϵ = area ratio
θ = conical half angle of nozzle, deg

Results

Equation (2) has been used for several mixture ratios of N_2O_4 − 50% UDMH + 50% hydrazine. The results are shown in Fig. 1. The chemical scale factor has been evaluated for a typical 20 K thrust engine using an effective experimental rate constant, $k = 6 \times 10^{15}$ $(\text{cm}^3/\text{mole})^2/\text{sec}$ obtained from Ref. 4 Results indicate that the atomic hydrogen reaction freezes last (freeze area ratio of 1.74 vs 1.35 for the OH radical, Fig. 2) and was used, therefore, in the remainder of the analysis. The nozzle flow is seen to freeze at area ratios of 1.74 and 1.16 for mixture ratios of 2.0 and 1.7, respectively.

The freeze point area ratio may be interpreted in terms of theoretical performance by cross plotting theoretical data for

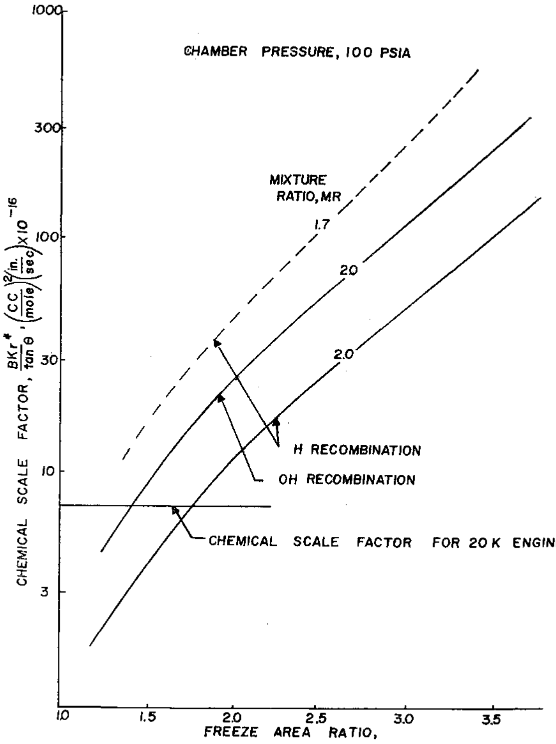

Fig. 1 Effective freeze area ratio location for a 20K thrust engine.

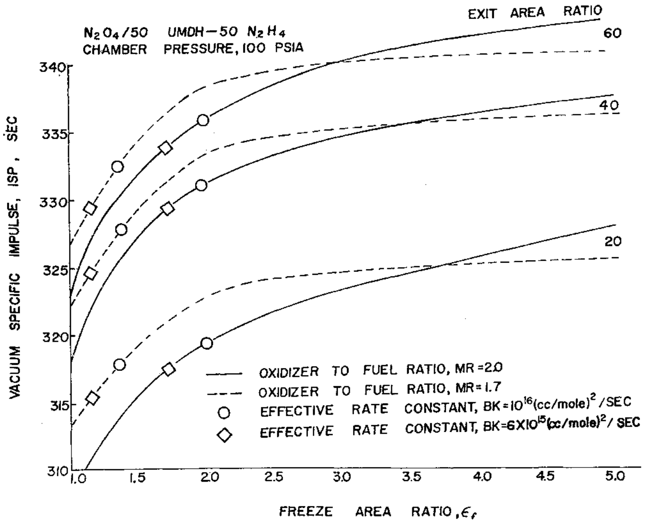

Fig. 2 Estimated theoretical vacuum specific impulse for a 20K thrust engine with chemical nonequilibrium.

Table 2 Specific impulse vs area ratio with kinetic losses

Exit area ratio	Mixture ratio, *MR*	I_{sp}, non-equilibrium	I_{sp}, theoretical shifting equilibrium
20	2.0	318	327
40	2.0	329	339
60	2.0	334	345
20	1.7	315	325
40	1.7	325	335
60	1.7	329	341

various freeze area ratios. These data are shown in Fig. 2. The estimated freeze area ratios from Fig. 1 also are shown for the rate coefficient 6×10^{15} (cm^3/mole)^2/sec at several large exit area ratios. The effects of increasing the rate constant to 10^{16}, the upper limit on rate coefficient, also are shown.

The salient results are as follows:

1) In the region of the throat the rate of change of impulse with freeze area ratio is large and the lower mixture ratio theoretical performance is greater than the higher mixture ratio.

2) When kinetic considerations are superimposed on this graph, the performance at the higher mixture ratio is always greater than the lower, regardless of small perturbations in geometry or rate constant.

3) The performance loss is higher for larger exit area ratios and lower mixture ratios.

Results of Fig. 2 are summarized in Table 2. Other losses, such as combustion, drag, divergence, etc., can be added to the recombination losses by known methods to predict delivered I_{sp}. Thus, the kinetic scheme shown here provides a powerful tool for predicting the performance of space engines using the presently most attractive storable propellant combinations.

References

[1] Bray, K. N. C., "Atomic recombination in a hypersonic wind tunnel nozzle," J. Fluid Mech. **6**, 1–32 (1959).

[2] Kushida, R., "An approximate method of analyzing non-equilibrium recombination effects in exhaust nozzles," Marquardt Corp. Rept. 20091 (March 3, 1960).

[3] Hall, J. G., "Dissociation non-equilibrium in hypersonic nozzle flows," Am. Inst. Chem. Engs. Preprint 7 (May 1959).

[4] Koppang, R. R., Bahn, G. J., and Giffoni, S., "Jet propulsion exhaust nozzle research program," Marquardt Corp. Rept. 25027 on BuWeps Contract as 59-6148-C (February 24, 1961).

[5] Hoglund, R., Carlson, D., and Byron, S., "Experiments on recombination effects in rocket nozzles," AIAA J. **1**, 324–329 (February 1963).

[6] Wray, K. L., "Chemical kinetics of high temperature air," *Hypersonic Flow Research*, edited by F. R. Riddell (Academic Press, New York, 1962), p. 181.

[7] Kaskan, W. E., "Excess radical concentrations and the disappearance of carbon monoxide in flame gases from some lean flames," Combust. Flame **3**, 49–56 (1959).

Combined External and Internal Cooling

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Nomenclature

A_s = cooled surface area
c_{pc} = specific heat of coolant
d = wall thickness
h_c = coolant-side heat-transfer coefficient
h_c' = defined by Eq. (8)
h_g = gas-side heat-transfer coefficient without film cooling
h_g' = gas-side heat-transfer coefficient with film cooling
k = thermal conductivity of the wall
Q/A = heat flux
T_{ad} = adiabatic wall temperature without film cooling
T_{ad}' = adiabatic wall temperature with film cooling
T_{c_0} = temperature of the coolant in coolant manifold
T_c = temperature of coolant at exit of film-coolant injector
T_{w_c} = coolant-side wall temperature
T_{w_g} = gas-side wall temperature
$\dot{w}_c$ = film-coolant flowrate
$\dot{w}_c^*$ = film-coolant flowrate when $h_c = 0$
ϵ = dimensionless measure of distance from film-coolant injector, where wall temperature $= T_c$
η = defined by Eq. (9)
η^* = defined by Eq. (6)
ϕ = efficiency factor
ψ = efficiency factor

WITH the extensive use of hydrogen as a propellant in rocket engines, a potentially important method of cooling is gaseous film cooling, whereby hydrogen, injected through holes or slots in the chamber wall, is interposed between the wall and the primary gas stream. Film cooling usually is considered when heat transfer is so intense that external (convection) cooling, normally called regenerative cooling, alone is not satisfactory because of a high pressure drop of the coolant. Because some loss of rocket engine performance occurs with film cooling, a pertinent question is raised concerning the effectiveness of a combination of the two methods as compared with the use of film cooling alone.

Analysis

Eckert[1] states that the heat-transfer coefficient with film cooling, h_g', should be defined as

$$h_g' \equiv (Q/A)/(T_{ad}' - T_{w_g}) \qquad (1)$$

where T_{ad}' is the adiabatic wall temperature with film cooling. Using this definition, experiments made in Refs. 2–4 indicated that, under most conditions of practical interest, there is little difference in the heat-transfer coefficients with and without film cooling. Equation 1 then can be rewritten as

$$h_g \simeq (Q/A)/(T_{ad}' - T_{w_g}) \qquad (2)$$

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